Exercise 29

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$G(t) = \frac{1-2t}{3+t}$$

Solution

The domain of $G(t)$ is

$$3 + t \neq 0$$

$$t \neq -3$$

$$\{t \mid t \neq -3\}.$$

Calculate the derivative of $G(t)$ using the definition.

$$\begin{aligned}
G'(t) &= \lim_{h\to 0} \frac{G(t+h) - G(t)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1-2(t+h)}{3+(t+h)} - \frac{1-2t}{3+t}}{h} \\
&= \lim_{h\to 0} \frac{\frac{1-2t-2h}{3+t+h} - \frac{1-2t}{3+t}}{h} \\
&= \lim_{h\to 0} \frac{\frac{(3+t)(1-2t-2h)}{(3+t)(3+t+h)} - \frac{(1-2t)(3+t+h)}{(3+t)(3+t+h)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{(3+t)(1-2t-2h)-(1-2t)(3+t+h)}{(3+t)(3+t+h)}}{h} \\
&= \lim_{h\to 0} \frac{(3+t)(1-2t-2h) - (1-2t)(3+t+h)}{h(3+t)(3+t+h)} \\
&= \lim_{h\to 0} \frac{(\cancel{3} - \cancel{6t} - 6h + t - \cancel{2t^2} - 2ht) - (\cancel{3} + t + h - \cancel{6t} - \cancel{2t^2} - 2th)}{h(3+t)(3+t+h)} \\
&= \lim_{h\to 0} \frac{(-6h + t - 2ht) - (t + h - 2th)}{h(3+t)(3+t+h)} \\
&= \lim_{h\to 0} \frac{-7h}{h(3+t)(3+t+h)} \\
&= \lim_{h\to 0} \frac{-7}{(3+t)(3+t+h)} \\
&= -\frac{7}{(3+t)^2}
\end{aligned}$$

The domain of $G'(t)$ is

$$(3+t)^2 \neq 0$$

$$3+t \neq 0$$

$$t \neq -3$$

$$\{t \mid t \neq -3\}.$$